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EVOLVING NOVEL COEFFICIENT SETS FOR OPTIMIZED RECONSTRUCTION OF QUANTIZED ONE-DIMENSIONAL (1-D) AND TWO-DIMENSIONAL (2-D) SIGNALS

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This paper describes a genetic algorithm that evolves optimized sets of coefficients for signal reconstruction under lossy conditions due to quantization. Beginning with a population of mutated copies of the set of coefficients describing a wavelet-based inverse transform, our genetic algorithm systemically evolves a new set of coefficients that significantly reduces mean squared error (relative to the performance of the selected wavelet) for various classes of one- and two-dimensional signals.

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EVOLVING NOVEL COEFFICIENT SETS FOR OPTIMIZED RECONSTRUCTION OF QUANTIZED ONE-DIMENSIONAL (1-D) AND TWO-DIMENSIONAL (2-D) SIGNALS

Abstract. This paper describes a genetic algorithm that evolves optimized sets of coefficients for signal reconstruction under lossy conditions due to quantization. Beginning with a population of mutated copies of the set of coefficients describing a wavelet-based inverse transform, our genetic algorithm systemically evolves a new set of coefficients that significantly reduces mean squared error (relative to the performance of the selected wavelet) for various classes of 1-D and 2-D signals.

1 INTRODUCTION

Wavelets [1] are commonly used to solve problems drawn from a wide range of application areas. Wavelet transforms associated with orthonormal, compactly supported wavelets [2] and biorthogonal wavelets [3] have been shown to achieve signal compression ratios as high as 10:1, 50:1, and even 100:1 without significant information loss. In these cases, the corresponding inverse wavelet transforms are capable of reconstructing very close approximations of the original signal.

For many practical problems, however, it becomes necessary to represent a given signal using a smaller range of possible values. For example, telephone signals (for which speed of transmission is most important) are represented by as few as 8 bits, while music signals (which require higher signal fidelity) are typically represented by 16-bit values. *Quantization* [4] is the process of mapping signals to a smaller number of bits. Figure 1 illustrates the process of reconstructing quantized 1-D signals. A 2-D transform is accomplished by performing a 1-D transform on each row of the image, followed by a 1-D transform on each column.

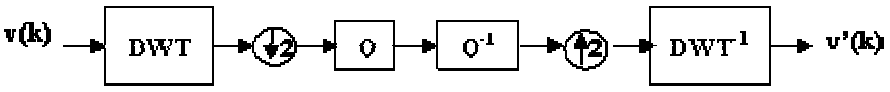

Figure 1. 1-D Reconstruction Discrete Wavelet Transform (DWT) Filter with Quantization

Errors introduced into the transformed signal via quantization may have an unacceptably adverse effect upon the quality of the signal when it is subsequently reconstructed via the wavelet inverse transform. A growing amount of empirical evidence (e.g., [5]) suggests that nontraditional inverse transformations may do a better job of compensating for the negative effects of quantization, resulting in higher fidelity signal reconstruction. In particular, recent studies suggest that the use of adaptive ([6], [7]) and/or nonstandard [8]

filters may significantly reduce errors for specific classes of signals, which typically share spatial domain characteristics.

2 METHODOLOGY

Genetic algorithms (GAs) are optimization techniques inspired by Darwinian evolution. GAs [9] have been successfully applied to an ever-increasing number of difficult and interesting optimization problems. The goal of this investigation was to develop a GA capable of automatically modifying the coefficient sets describing wavelet inverse transform functions [10] to evolve novel inverse transforms exhibiting significantly improved performance for a given class of signals [11]. In particular, our GA automatically compensated for errors introduced into the original signal by quantization. Performance may be measured in many ways [12]; for this study, performance equaled the mean squared error (MSE) in the reconstructed signal.

Figure 2 illustrates our GA-based inverse transform optimization process. The best-of-run inverse transform coefficients produced by the GA are used to replace the DWT^{-1} shown in Figure 1. To improve upon wavelet-based techniques, our GA had to evolve optimized inverse transform coefficients that significantly reduced the aggregate MSE in each reconstructed signal v'(k).

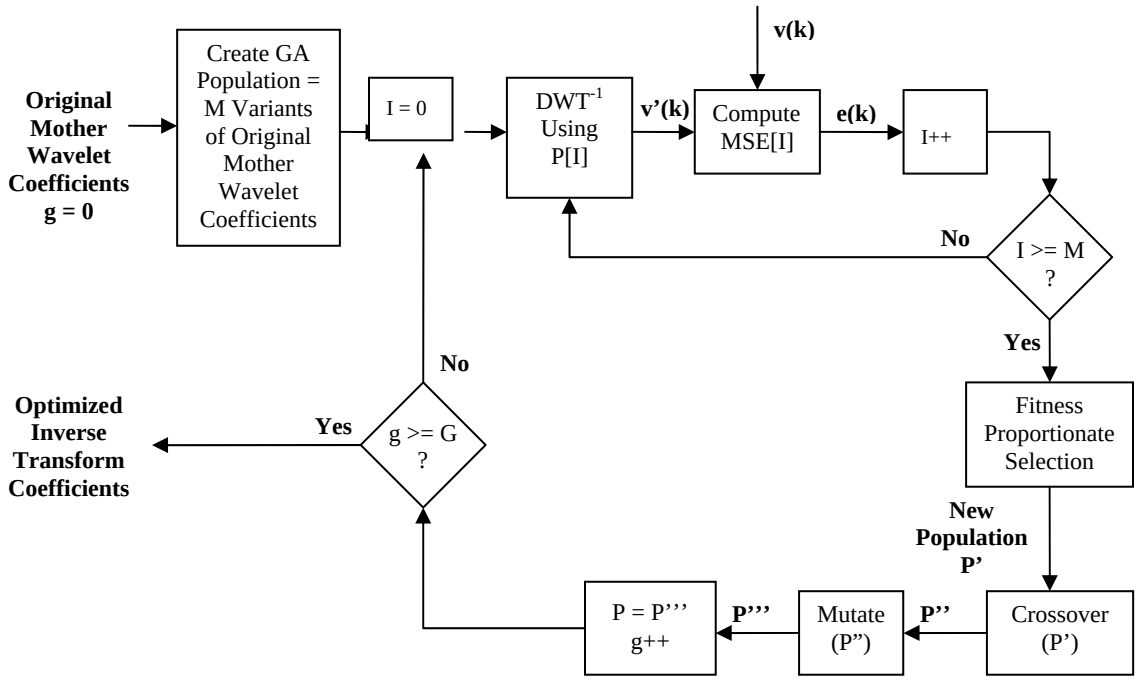

Figure 2. Detailed GA for Optimization of Coefficients for Inverse Transforms

3 TEST RESULTS

We conducted a series of tests to demonstrate the performance of GA-evolved transforms relative to that of the *Daubechies-4* (Daub4) wavelet inverse transform [10], which may be described by the following sets of floating-point coefficients:

h2 = {0.4830, 0.8365, 0.2241, -0.1294}
g2 = {-0.1294, -0.2241, 0.8365, -0.4830}

Each test was characterized by a particular combination of the following parameters:

1. **SIGNAL CLASS**. Each test trained our GA using signals drawn from a particular class. One-dimensional signal classes used in this study included ramp functions, sine waves, and random signals. Additional tests were performed using 2-D photographic images. These classes were chosen due to their relevance to real-world applications.
2. **G** specified the (maximum) number of generations executed by our GA. Preliminary experiments indicated that a large G value was necessary to allow our GA to progress toward a globally optimized solution. For each of the 1-D signal tests performed for this study, G = 10,000. Since the amount of data processed during the 2-D tests was an order of magnitude greater than for 1-D signals, it became necessary to use only G = 500 generations.
3. **M** specified the population size (i.e., the number of candidate solutions in each generation). For each of the tests performed for this study, M = 500.
4. $\mathbf{P_C}$ specified the percentage of individuals in the next generation subjected to the crossover operator. Preliminary experiments demonstrated the benefits of a high crossover rate; for this reason, each of the tests performed for this study used P_C = 100 percent * (M-1)/M = 99.8 percent.
5. $\mathbf{P_M}$ specified the probability of mutation, as described below.
6. **N** specified the size of the training population (i.e., the number of signals used to train our GA).

Our GA copied the best individual from the current generation into position 0 of the next generation; this individual was not subject to subsequent crossover or mutation. Our GA selected the remaining M-1 individuals from the current generation via tournaments of three randomly selected individuals. These individuals were then probabilistically subjected to the crossover operator according to P_C. Finally, each coefficient of these M-1 individuals was subjected to the mutation operator with probability P_M. Our GA initialized P_M to 2 percent. If the current generation failed to produce a new globally optimal set of coefficients, our GA automatically increased P_M by 1 percent, up to a maximum of 20 percent; otherwise, our GA reset P_M to 2 percent.

3.1 Class 1: Ramp Signals

One-dimensional ramp signals are important for a variety of applications. For example, ramp response signals provide information about the size, shape, and orientation of dielectric targets [13]. Tests 1, 2, and 3 populated the training set with N ramp signals,

each of which contained 50 values. For each signal, our GA initialized the starting value **α** to a randomly selected integer between 255 and 511; set the stopping value **ω** to **α**/2; and used the following algorithm to determine the remaining values:

```
if (v[i] == ω)
        v[i+1] = α;
else
        v{i+1] = v[i] – 1;
```

Our GA initialized each of the g2 and h2 coefficients for the inverse transforms in generation 0 to a randomly mutated version of the corresponding coefficient from the Daub4 wavelet.

Table 1 summarizes the performance (as measured by the total MSE for N reconstructed signals) of each novel inverse transform described by a GA-evolved best-of-run coefficient set, relative to that of the Daub4 wavelet.

Table 1. Ramp Signal Test Results

Test	N	MSE (Daub4)	MSE (evolved)	% Improvement
1	100	5505.1	4838.9	12.1
2	25	1376.0	1215.0	11.7
3	10	552.4	522.9	5.35

These tests evolved the following sets of coefficients for the optimized inverse transform:

Test 1: h2 = {0.3167, 0.8038, 0.3911, -0.0979}
g2 = {-0.1700, -0.6120, 0.9813, -0.3859}
Test 2: h2 = {0.3203, 0.8113, 0.3875, -0.1054}
g2 = {0.1493, -0.0852, -0.4700, 0.2132}
Test 3: h2 = {0.4053, 0.8110, 0.3024, -0.1052}
g2 = {-0.0705, 0.1193, -0.8903, 0.2180}

The results of these tests demonstrate the following key points:

1) Novel inverse transforms exist that outperform the Daub4 wavelet inverse transform for reconstructing arbitrary ramp signals that have been subjected to lossy quantization operations.
2) Our GA is capable of automatically optimizing coefficient sets for these novel inverse transforms.
3) Training on a larger population of ramp signals allows our GA to evolve solutions that exhibit better generalization properties.

3.2 Class 2: Sine Waves

The detection of periodic behavior in 1-D signals continues to be a research topic of considerable importance [14]. Tests 4 through 12 populated the training set with various types of sine waves. Each wave was characterized by the following parameters:

g = gain
f = frequency
d = offset.

For these tests, each vector **v** consisted of 50 sampled values defined as follows:

```
for (i = 0; i < 50; i++)
    v[i] = g * sin (2πf * i) + d;
```

Table 2 summarizes the performance of GA-evolved best-of-run coefficient sets optimized under conditions described by various combinations of frequency (**f**), gain (**g**), and offset (**d**), relative to that of the Daub4 wavelet described above.

Table 2. Sine Wave Test Results

Test	N	**g**	**d**	**f**	MSE (Daub4)	MSE (evolved)	% Improvement
4	100	128	256	0...999	1096.19	86.69	92.1
5	25	128	256	0...999	269.00	20.18	92.5
6	10	128	256	0...999	124.55	3.55	97.2
7	100	128	0...255	1000	5207.62	5034.13	3.33
8	25	128	0...255	1000	1203.68	1182.69	1.74
9	10	128	0...255	1000	403.67	397.82	1.45
10	100	0...127	256	1000	1123.89	88.19	92.2
11	25	0...127	256	1000	274.49	24.90	90.9
12	10	0...127	256	1000	108.94	10.30	90.6

These tests evolved the following sets of coefficients for optimized inverse transforms:

Test 4: h2 = {0.4875, 0.8279, 0.2109, -0.1294}
g2 = {-0.1280, -0.2430, -0.6366, -0.4881}
Test 5: h2 = {0.4876, 0.8280, 0.2109, -0.1294}
g2 = {0.1156, 0.2664, 0.9523, 0.3452}
Test 6: h2 = {0.4780, 0.8282, 0.2196, -0.1305}
g2 = {-0.1398, 0.2286, 0.8616, 0.4919}
Test 7: h2 = {0.4447, 0.7768, 0.2569, -0.0747}
g2 = {-0.1050, -0.3507, -0.2918, 0.3030}
Test 8: h2 = {0.4448, 0.7849, 0.2592, -0.0805}
g2 = {0.0752, -0.2607, -0.7085, -0.6749}
Test 9: h2 = {0.4927, 0.8024, 0.2175, -0.0920}
g2 = {-0.0649, 0.5076, -1.0222, 0.2395}
Test 10: h2 = {0.4877, 0.8278, 0.2106, -0.1294}
g2 = {-0.1631, -0.2723, -0.9434, 0.5711}
Test 11: h2 = {0.4876, 0.8280, 0.2108, -0.1294}
g2 = {0.1542, -0.2062, -0.7879, 0.4552}
Test 12: h2 = {0.4777, 0.8279, 0.2208, -0.1293}
g2 = {0.1337, -0.2792, 0.7655, -0.2608}

Tests 4, 5, and 6 demonstrated that our GA was capable of identifying coefficients for inverse transforms that significantly outperformed the Daub4 inverse transform for the task of reconstructing sine waves characterized by different **f** values. For this signal class, GA-optimized transforms were capable of reducing MSE in the reconstructed signal by a factor of 12 or more. It is interesting to note that, while the magnitude of the four coefficients comprising set h2 were virtually identical, the four coefficients from set g2 exhibited far greater variation, even to the extent of having opposite signs.

Tests 7, 8, and 9 showed that, when the training set consisted of sine waves that differed only according to the offset **d**, little advantage was to be gained from evolving novel coefficients for inverse transforms. For these three tests, the performance of the inverse transforms described by the evolved coefficient sets improved upon that of the Daub4 inverse transform by an average of only 2.17 percent. These tests also showed much greater variation in the magnitude of g2 coefficients than h2 coefficients, as well as differences in the sign of those coefficients.

Tests 10, 11, and 12 demonstrated our GA's ability to evolve coefficients for inverse transforms that significantly outperformed the Daub4 inverse transform for the task of reconstructing sine waves that differed only in the gain value **g**. For this class of signal, GA-optimized inverse transforms were capable of reducing MSE in the reconstructed signal by a factor of 11 or more. As with previous tests, these results showed greater variation in the magnitude of g2 coefficients; indeed, the h2 coefficients evolved during these three tests were identical in sign and virtually identical in magnitude, while the g2 coefficients varied in both sign and magnitude.

3.3 Class 3: Random Functions

Tests 13, 14, and 15 populated the training set with one-dimensional vectors **v** consisting of 50 random values between 0 and 255 (inclusive). Table 3 summarizes test results under a variety of conditions, relative to the MSE produced by the Daub4 inverse wavelet.

Table 3. Test Results for Random Signals

Test	N	MSE (Daub4)	MSE (evolved)	% Improvement
13	100	12680.4	12529.2	1.19
14	25	3271.3	3230.7	1.24
15	10	1390.0	1333.7	1.04

These tests evolved the following sets of optimized inverse transform coefficients:

Test 13: h2 = {0.4781, 0.8248, 0.2285, -0.1177}
g2 = {-0.1279, -0.2174, 0.8248, -0.4728}
Test 14: h2 = {0.4781, 0.8268, 0.2331, -0.1209}
g2 = {-0.1236, -0.2262, 0.8199, -0.4763}
Test 15: h2 = {0.4864, 0.8282, 0.2234, -0.1192}
g2 = {-0.1363, -0.2135, 0.8263, -0.4734}

The average performance (in terms of MSE) of the inverse transforms described by our GA-evolved coefficient sets from tests 13, 14, and 15 was only 1.16 percent better than that of the Daub4 wavelet inverse transform. This advantage remained nearly negligible as the number of random signals in the training population increased. Truly random signals exhibit no particular pattern; for this reason, our GA was unable to find and exploit information common to all of the signals in the training set.

3.4 Class 4: Images

The goal of Tests 16 and 17 was to determine whether the GA-based methodology established by Tests 1-15 could be used to evolve a transform that outperformed the Daub4 wavelet when reconstructing *2-D images* previously subjected to quantization error. Test 16 used a training set populated with N portrait-like photographs similar to the image shown in Figure 3, while Test 17 used landscape-like photographs similar to those shown in Figure 4. Each photographic image consisted of a 512- by 512-array of red, green, and blue (RGB) color pixels. Prior to evolution, our GA transformed each image from the training set to the corresponding signal in the luminance (YUV) domain. Each test used a quantization step of 64 (in effect, discarding the information contained in the six least significant bits of each value).

The results of Tests 16 and 17 are summarized in Table 4. These results clearly demonstrate that our GA was capable of evolving a best-of-run transform whose

performance, in terms of reduced total MSE over a class of images, measurably improved upon that of the Daub4 wavelet.

Table 4. Test Results for 2-D Images

Test	N	MSE (Daub4)	MSE (evolved)	% Improvement
16	5 (barb, baboon, lenna, susie, zelda)	2.30381e+08	2.18978e+08	4.95
17	5 (airplane, boat, fruits, goldhill, park)	1.85457e+08	1.78302e+08	3.86

Figure 3. A Typical Portrait-like Photographic Image ("barb.bmp") from Test 16

Figure 4. A Typical Landscape-like Photographic Image ("goldhill.bmp") from Test 17

These tests evolved the following sets of coefficients for optimized inverse transforms:

Test 16: h2 = {0.4686, 0.8035, 0.2378, -0.1021}
g2 = {-0.2084, -0.1461, 0.7251, -0.3935}
Test 17: h2 = {0.4637, 0.8116, 0.2449, -0.1095}
g2 = {-0.1680, -0.1644, 0.7555, -0.3732}

Considerable computational resources were necessary to complete each 2-D run. For example, Test 17 required 46 hours, 53 minutes, 59 seconds of wall clock time on a high-end dedicated PC. Access to supercomputers would allow future tests to use much larger populations (e.g., M = 10,000) over many more generations (e.g., G = 5,000), possibly evolving inverse transforms that exhibit substantially better reconstruction properties than those described above.

4 CONCLUSIONS

Collectively, the results of this study suggested that the number of coefficient sets capable of producing high-fidelity signal reconstructions under lossy conditions may be much larger than previously believed. The novel coefficient sets evolved during this study violated wavelet properties required for perfect reconstruction, such as invertibility

and nonredundancy [15]. Nevertheless, the corresponding inverse transforms consistently outperformed the Daub4 inverse transform, often producing significantly higher fidelity reconstructions of periodic signals and images, as measured by the percentage reduction in the MSE of each reconstructed signal. The results of this study strongly encouraged the identification and use of evolved inverse transforms for signal reconstruction under lossy conditions. In particular, our results conclusively demonstrated our GA's ability to automatically identify novel sets of coefficients for inverse transforms that successfully reconstruct various classes of periodic signals and images under lossy conditions subject to quantization.

Our research will continue with significantly larger tests for multidimensional signals and will include a rigorous mathematic analysis of our findings.

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